

MILLISECOND PULSAR EMISSION ALTITUDE FROM RELATIVISTIC PHASE SHIFT: PSR J0437-4715

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ABSTRACT

We have analyzed the profile of the millisecond pulsar PSR J0437-4715 at 1440 MHz by fitting the Gaussians to pulse components, and identified its 11 emission components. We propose that they form a emission beam with 5 nested cones centered on the core. Using the phase location of component peaks, we have estimated the aberration–retardation (A/R) phase shift. Due to A/R phase shift, the centroid of intensity profile and the inflection point of polarization angle swing are symmetrically shifted in the opposite directions with respect to the meridional plane, which is defined by the rotation and magnetic axes. By recognizing this fact, we have been able to locate the phase location of meridional plane and estimate the absolute altitude of emission of core and conal components relative to the neutron star center. Using the more exact expression for phase shift given recently by Gangadhara (2005), we find that the radio emission comes from a range of altitude starting from the core at 7% of light cylinder radius to outer most cone at 30%.

Subject headings: pulsars: general —pulsars: individual (PSR J0437-4715)

1. INTRODUCTION

The shape and polarization of integrated pulse of a pulsar are quite stable, and they are expected to reflect the magnetic field geometry and the structure of pulsar emission beam. The common occurrence of odd number of components and their distribution across the pulse window has led to the nomenclature of core and cone structure for the pulsar emission beam (Rankin 1983a). However, Lyne & Manchester (1988) and Manchester (1995) have argued for a different interpretation, in which components on either side of core arise from a random distribution of emitting regions, which lead to the so-called patchy cone.

Mitra & Deshpande (1999) have attempted to test these models, and found the evidences in favor of conal structure. The frequency evolution of profile structure and polarization angle swing of pulsars has been interpreted by considering the radio emission by relativistic plasma moving in dipolar magnetic field lines (e.g., Radhakrishnan & Cooke 1969; Sturrock 1971; Ruderman & Sutherland 1975; Rankin 1983a&b, 1990, 1993; Lyne & Manchester 1988; Blaskiewicz et al. 1991; Hibschan & Arons 2001; Mitra & Li 2004).

For understanding the pulsar radio emission mechanism, the radio emission altitude bears an at most importance. There have been two kinds of methods proposed for estimating the radio emission altitudes: (1) *Purely geometric method*, which assumes the pulse edge is emitted from the last open field lines (e.g., Cordes 1978; Gil & Kijak 1993; Kijak & Gil 2003), (2) *Relativistic phase shift method*, which assumes the asymmetry in the conal components phase location, relative to core, is due to the aberration-retardation phase shift (e.g., Gangadhara & Gupta 2001, hereafter GG01). Both the methods have merits and demerits: former method has an ambiguity in identifying the last open field lines while the later rely on the clear identification of core-cone structure of the emission beam. Thomas & Gangadhara (2005) have showed that the profiles becomes asymmetric due to the influence of rotation on the radio emitting sources moving along the dipolar magnetic field lines.

By assuming the pulsar radio emission comes from fixed altitude, Blaskiewicz et al. (1991) have showed that polarization angle inflection point (PAIP) lags with respect to the centroid of the intensity profile by $\sim 4r/r_{LC}$, where r is the emission altitude measured from the center of neutron star and $r_{LC} = Pc/2\pi$ is the light cylinder radius. The parameters P and c are the pulsar spin period and speed of light, respectively. The results of purely geometric method are found to be in rough agreement with those of Blaskiewicz et al. (1991). However, the relativistic phase shift method clearly indicates that the emission altitude across the pulse window is not constant (GG01; Gupta & Gangadhara 2003, hereafter GG03; Johnston & Weisberg 2006). Dyks, Rudak and Harding (2004, hereafter DRH04) by revising the relation for aberration phase shift given in GG01, have re-estimated the emission heights. The average emission heights are found to be comparable with those of geometric method.

By considering the relativistically beamed radio emission in the direction of magnetic field line tangents, Gangadhara (2004, hereafter G04) has solved the viewing geometry in an inclined and rotating dipole magnetic field. He has showed that due to geometric restrictions in receiving the beamed emission, a distant observer can not receive the radio waves from a constant altitude. A more exact expression for relativistic phase shift (A/R) is given by Gangadhara (2005; hereafter G05), which includes the phase shift due to polar cap currents also. Under small angle approximations, the expression of G05 reduces to that of DRH04.

Recently, Johnston & Weisberg (2006) have estimated the emission heights of the young

pulsar PSR J1015-5719 using the expression for A/R phase shift given in G05. In this paper, we intend to estimate the emission heights of the nearby, bright millisecond pulsar PSR J0437-4715. It was discovered in the Parkes southern survey (Johnston et al. 1993; Manchester et al. 1996), has a pulse period of 5.75 ms. Navarro et al. (1997) have presented the mean pulse observations of PSR J0437-4715 at multiple radio frequencies, and found a frequency dependent lag of the total-intensity profile with respect to the polarization profile. Jenet et al. (1998) have presented the results based on the high time resolution single pulse observations of this pulsar. They find the single pulse properties of PSR J0437-4715 are similar to those of the common slow-rotating pulsars.

The outline of this paper is as follows. In §2, we discuss a method for identifying the phase location of meridional plane. Next, in §3 we consider the mean pulse data of millisecond PSR J0437-4715, and fit the Gaussian to pulse components. By assuming conal emission beam, we have estimated the A/R phase shifts and the emission heights.

2. Phase of meridional plane

In the nested cone structure of pulsar emission beam, the conal components are expected to be distributed symmetrically on either side of core when observed in the corotating frame. However, for an observer in the laboratory frame the profile becomes asymmetric due to rotation. A typical asymmetric profile is given in Figure 1, and such an asymmetry in the real profiles has been reported for many pulsars (e.g., GG01; GG03; Johnston & Weisberg 2006).

Blaskiewicz et al. (1991) have generalized the rotating vector model (RVM) by considering the first order relativistic effects, which arise due to the pulsar rotation. By assuming that the emission across the entire pulse comes from a constant height, they have showed that the centroid of the total intensity pulse advances to earlier phase by $\sim r/r_{\text{LC}}$ while the polarization angle inflection point (PAIP) is delayed to later phase by $\sim 3r/r_{\text{LC}}$, where r is the emission height measured from the center of neutron star. However, there are clear evidences to show that the emission do not come from a fixed altitude (GG01; GG03; Johnston & Weisberg 2006). Consider the intensity profile given in Figure 1, in which, C_l and C_t are the components of an arbitrary cone with center at CC. If the emission altitude is not constant across the pulse window then the retardation also contributes to the phase shift, and it is comparable to that of aberration (G05). So, due to A/R effects, the centroid intensity profile is advanced to earlier phase by $\sim 2r/r_{\text{LC}}$ while the corresponding PAIP is delayed to later phase by $\sim 2r/r_{\text{LC}}$ (e.g., DRH04; Fung 2005). Hence, the cone center CC and PAIP lie symmetrically on either side of meridional plane M, which is at the phase

$\phi' = 0^\circ$. The meridional plane is defined by the plane containing the rotation axis and the magnetic axis. Note that the phase location of M is invariant with respect to r . In other words, both CC and PAIP come closer to M at smaller r , and move away from it at larger r . In the co-rotating frame, i.e., in the absence of rotation, the A/R effects vanish, and hence core peak and PAIP appear at the meridional plane M.

The field aligned polar cap current do not introduce any significant phase shift into the phase of PAIP but it does introduce a positive offset into polarization angle, however, it roughly cancels due to the negative offset by aberration (Hibschman & Arons 2001). The phase shift of pulse components due to polar cap current is estimated recently by G05, and found to be quite small compared to A/R phase shift.

3. Emission height in millisecond pulsar: PSR J0437-4715

Consider the nearest bright millisecond pulsar PSR J0457-4715, which has a period of 5.75 ms. It was discovered in Parkes southern survey by Johnston et al. (1993), and is in a binary system with white dwarf companion. Manchester & Johnston (1995) have presented the mean pulse polarization properties of PSR J0437-4715 at 1440 MHz. There is significant linear and circular polarization across the pulse with rapid changes near the pulse center. The position angle has a complex swing across the pulse, which is not well fitted with the rotating vector model, probably, due to the presence of orthogonal polarization modes. The mean intensity pulse has a strong peak near the center, where the circular polarization shows a clear sense reversal and the polarization angle has a rapid sweep. These two features strongly indicate that it is a core. The profile shows more than 8 identifiable components.

Table 1. Parameters related to radio emission from PSR J0437–4715 at 1440 MHz

Cone No.	ϕ_1'	ϕ_t'	$\delta\phi'$	Γ		r		Δ	s/s_{lof}
	($^{\circ}$)	($^{\circ}$)	($^{\circ}$)	($^{\circ}$)	(Km) ^a	(Km) ^b	(% r_{LC})	($^{\circ}$)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	—	—	−9.50±0.26	4.00±0.00	22.8	20.3±0.6	7	12	0.17±0.00
1	−29.50±0.24	8.50±0.46	−10.50±0.26	7.06±0.06	25.2	23.3±0.5	9	8	0.28±0.01
2	−50.00±0.50	21.50±0.54	−14.25±0.37	11.50±0.10	34.2	34.3±0.9	13	0	0.38±0.01
3	−81.50±0.15	37.94±0.08	−21.78±0.08	18.00±0.02	52.2	64.3±0.3	23	19	0.43±0.00
4	−102.01±0.36	56.14±0.29	−22.90±0.23	22.93±0.05	55.0	84.9±0.9	31	35	0.47±0.00
5	−129.50±1.01	89.54±4.15	−19.98±2.10	29.30±0.40	48.0	91.8±6.2	33	49	0.57±0.02

^aEmission heights computed from the approximate formula of DRH04, and ^bthose using the more exact formula of G05.

Consider the average pulse profile given in Figure 2 for PSR J0437-4715 at 1440 MHz. To identify pulse components and to estimate their peak locations we followed the procedure of Gaussian fitting to pulse components. Two approaches have been developed and used by different authors. Unlike Kramer et al. (1994), who fitted the *sum* of Gaussians to the total pulse profile, we separately fitted a single Gaussian to each of the pulse components. We used the package Statistics‘NonlinearFit’ in Mathematica for fitting Gaussians to pulse components. It can fit the data to the model with the named variables and parameters, return the model evaluated at the parameter estimates achieving the leastsquares fit. The steps are as follows: (i) Fitted a Gaussian to the core component (VI), (ii) Subtracted the core fitted Gaussian from the data (raw), (iii) The residual data was then fitted for the next strongest peak, i.e., the component IV, (iv) Added the two Gaussians and subtracted from the raw data, (v) Next, fitted a Gaussian to the strongest peak (IX) in the residual data. The procedure was repeated for other peaks till the residual data has no prominent peak above the off pulse noise level.

By this procedure we have been able to identify 11 of its emission components: I, II, III, IV, V, VI, VII, VIII, IX, X and XI, as indicated by the 11 Gaussians in Figure 2. The distribution of conal components about the core (VI) reflect the core-cone structure of the emission beam. We propose that they can be paired into 5 nested cones with core at the center.

Manchester & Johnston (1995) have fitted the rotating vector model (RVM) of Radhakrishnan & Cooke (1969) to polarization angle data of PSR J0437-4715 and estimated the polarization parameters $\alpha = 145^\circ$ and $\zeta = 140^\circ$. The parameter α is the magnetic axis inclination angle relative to the pulsar spin axis, and $\zeta = \alpha + \beta$, where β is the sight line impact angle relative to the magnetic axis. However, for these values of α and ζ the colatitude s/s_{lof} (see eq. [15] in GG01) exceeds 1 for all the cones. The parameter s gives the foot location of field lines, which are associated with the conal emissions, relative to magnetic axis on the polar cap. It is normalized with the colatitude s_{lof} of last open field line. However, Gil and Krawczyk (1997) have reported $\alpha = 20^\circ$ and $\beta = -4^\circ$ by fitting the average intensity pulse profile rather than by formally fitting the RVM. Further, they find the relativistic RVM by Blaskiewicz et al. (1991) calculated with these α and β seems to fit the observed position angle quite well than the non-relativistic RVM. Further, Gil and Krawczyk (1997) have reported that the centroid of intensity profile and PAIP are separated by $\sim 40^\circ$. By taking the cone center (CC) of outermost cone as the centroid of intensity profile, we can locate the phase of meridional plane M. It is at the mid point, i.e., at $\delta\phi' \sim 20^\circ$ between CC and PAIP. Using $\delta\phi' \approx 2r/r_{\text{LC}}$, we get the emission height $r \approx 48$ km. We have adopted these values of α and β in our model as they also confirm $s/s_{\text{lof}} \leq 1$ (see below). The phase $\phi' = 0^\circ$, in Figure 2, corresponds to the phase of meridional plane (M), which is at the mid point

between CC of outermost cone and PAIP of relativistic polarization angle swing.

In column 1 of Table 1 we have given the cone numbers. The cone number 0 stands for core component. The peak locations of conal components on leading and trailing sides are given in columns 2 and 3, respectively. The conal components are believed to arise from the nested hollow cones of emission (Rankin 1983a, b, 1990, 1993), which along with the central core emission, make up the pulsar emission beam. Let ϕ'_l and ϕ'_t be the peak locations of conal components on leading and trailing sides of a pulse profile, respectively. Then, using the following equations, we estimate the phase shift ($\delta\phi'$) of cone center with respect to M and the phase location (ϕ') of component peaks in the absence of phase shift, i.e., in the corotating frame (see eq. [11] in GG01):

$$\delta\phi' = \frac{1}{2}(\phi'_t + \phi'_l) , \quad \phi' = \frac{1}{2}(\phi'_t - \phi'_l) . \quad (1)$$

In column 4 of Table 1, we have given the values of $\delta\phi'$. It increases in magnitude from inner most cone to 4th cone, but for outermost cone i.e., for 5th cone it decreases slightly. We suspect this decrease in $\delta\phi'$ for the 5th cone is due to the fact that emission spot moves closer to the rotation axis, and hence aberration phase shift becomes smaller (G05).

The half-opening angle Γ (see eq. [7] in G04) of the emission beam is given in column 5, and it is about 30° for the last cone. In column 6, we have given the emission heights computed from the approximate expression given in DRH04. Using the more exact formalism for A/R phase shift given in G05 (see eq. [45]), we computed the emission heights and given in column 7. Their percentage values in r_{LC} are given in column 8. It shows the emission in PSR J0437-4715 occurs over a range of altitude starting from the core at 20 Km to the outermost cone at 90 Km. In column 9, we have given the refinement $\Delta(\%) = (1 - c_6/c_7)100$, where c_6 and c_7 are the emission heights given in columns 6 and 7, respectively. It shows the refinement is quite significant for the outer cones. In column 10, we have given the colatitude s/s_{lof} of foot field lines, which are associated with the component emissions. It shows due to the relativistic beaming and geometric restrictions, observer tends to receive the emissions from the open field lines which are located in the colatitude range of 0.2 to 0.6 on the polar cap.

4. DISCUSSION

A long standing question in pulsar astronomy has been the location of the radio emission region in magnetosphere. The emission heights of PSR B0329+54 given in GG01, six other pulsars in GG03 and the revised ones in DRH04 are all relative to the emission height of core. They assumed that the core emission height is negligible compared to the light cylinder

radius of classical pulsars, but it may not be so in the case of millisecond pulsars. Though Blaskiewicz et al. (1991) have assumed a constant emission height in their model, they have indeed indicated the possibility of core emission at a lower altitude than conal emissions. We identify, due to A/R phase shift, that the meridional plane (M) is located at the mid point between the phase of core peak and polarization angle inflection point (PAIP). By recognizing this fact, we have been able to estimate the *absolute* emission heights of core as well as conal components. We find that the core emission in PSR J0437–4715 is indeed come from a lower altitude than the conal emissions. Note that our estimates of error bar do not include the error bars in α and β , as they are not reported. For comparison we have reported the emission heights (see column 6 in Table 1) estimated from the approximate relation of DRH04, which is independent of the parameters α and β . In the case of core and inner cones (1 and 2), the refinement in the emission height is $< 12\%$ (see column 9).

We selected PSR J0437-4715 because of its strong core and clear conal emissions. Polarization angle (PA) is complicated near the core component but we can still locate the meridional plane by avoiding fitting that part of PA data. The idea is that without the knowledge of polarization angle data near the core it is possible to locate the meridional plane by just fitting the relativistic polarization angle formula of BCW91 to segments of PA angle data corresponding to the conal components. Since the emission altitude for each cone is different, one can not fit the entire PA data with a constant altitude. If we can best fit a segment of PA data corresponding to a outer cone or any other inner cone it is possible to locate the phase of the meridional plane. Since we used the formula of Blaskiewicz et al. (1991), in which the aberration phase shift is independent of phase, our results are approximate.

The polarization angle curve obtained by fitting the segments of PA data under C_l and C_t gives a virtual inflection point (PAIP) corresponding certain height reported by fitting routine. Since the emission height vary for each cone, we shall have as many number of virtual inflection points as the cones. The point to be noted is that the cone center and PA virtual inflection point of each cone is approximately symmetrically located on either side of the meridional plane. Since the cone centers and the virtual PA inflection points are approximately symmetrically located on either side of meridional plane, it is possible to locate the phase of meridional plane even if we are unable to detect virtual PA inflection point for certain cones or core.

The geometrical method, which by comparing the measured pulse widths with geometrical predictions from dipolar models, can yield absolute emission heights. But such absolute values are likely to be underestimated due to an ambiguity in identifying the last open field lines. The profile widths generally measured at a 10%-level can give the lower limits for

emission altitudes (Xilouris et al. 1996). In PSR J0437–4715 the components I and XI are at the intensity level of 1 to 2%, and the field lines associated with them are not last open field lines, but they are found to be located at the colatitude $s/s_{\text{lof}} \sim 0.6$ on polar cap.

It is possible that the A/R phase shift can be reduced by the rotational distortion of magnetic field sweep back of the vacuum dipole magnetic field lines. It was first considered in detail by Shitov (1983). Dyks and Harding (2004) have investigated the rotational distortion of pulsar magnetic field by making the approximation of a vacuum magnetosphere. For $\phi' = 110^\circ$, $\beta = -4^\circ$ and $\alpha = 20^\circ$ we computed the phase shift due to magnetic field sweep back $\delta\phi'_{\text{mfsb}}$ (see eq. [49] in G05). But it is found to be $< 0.6^\circ$ for $r/r_{\text{LC}} \leq 0.3$, which is much smaller than the aberration, retardation and polar cap current phase shifts in PSR J0437–4715.

5. CONCLUSION

We have analyzed the mean profile of the millisecond PSR J0437-4715 at 1440 MHz, and identified 11 of its emission components. We propose that they form a emission beam consist of 5 nested cones centered on the core. Using the phase location of component peaks, we have estimated the aberration–retardation (A/R) phase shift. Due to A/R phase shift, the center of each cone and the inflection point of corresponding polarization angle swing are symmetrically shifted in the opposite directions with respect to the meridional plane. By recognizing this fact, we have been able to locate the phase location of meridional plane and estimate the absolute altitudes of emission of core and conal components relative to neutron star center. We used the more exact expression for phase shift given recently by G05. The radio emission at 1440 MHz in PSR J0437-4715 is found to come from an altitude range starting from the core at 7% of light cylinder radius to the outer most cone at 30%.

We used the data available on EPN archive maintained by MPIfR, Bonn. We are thankful to M. N. Manchester and S. Johnston for making their data available on web.

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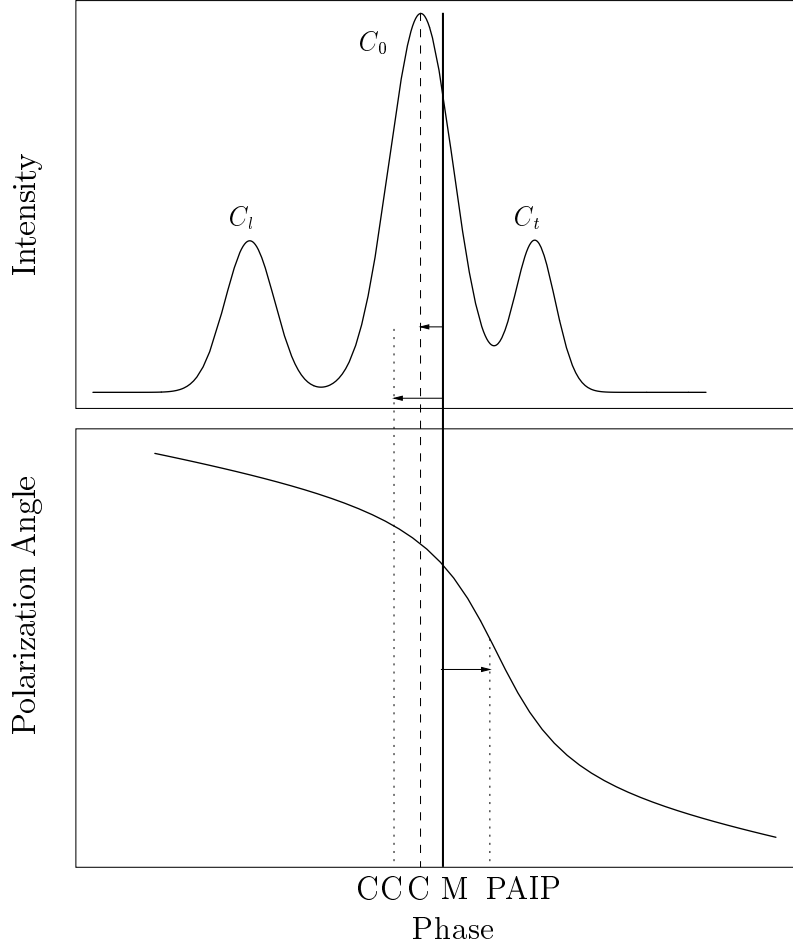


Fig. 1.— A typical pulse profile. C_0 is the core. C_l and C_t are the components on leading and trailing sides, respectively. The dotted lines represent the phase of cone center (CC) and polarization angle inflection point (PAIP). The thick line mark the phase of meridional plane (M), and the broken line mark the phase of core peak (C). Arrows indicate direction of phase shift of CC, C and PAIP with respect to M.

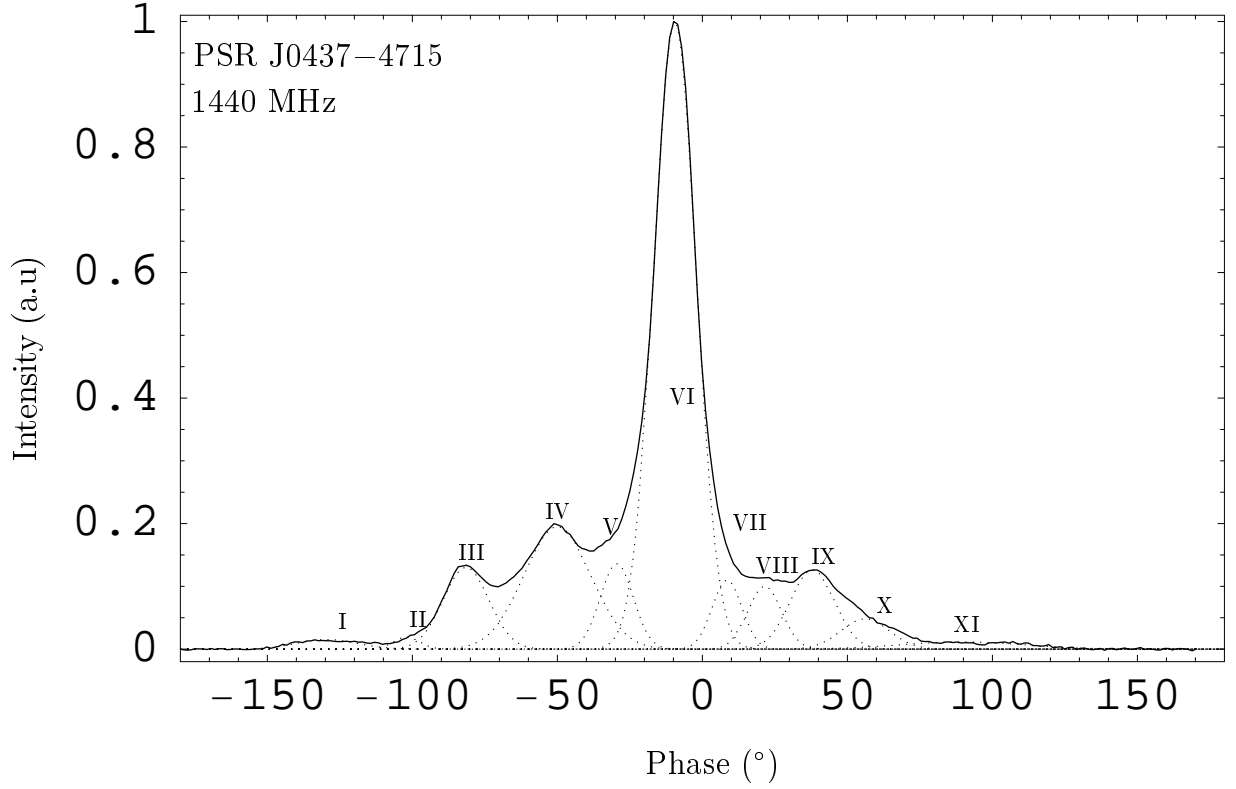


Fig. 2.— Average pulse of PSR J0437-4715 at 1440 MHz and the model Gaussians (dotted line curves) fitted to emission components. Due to A/R effects, the core (VI) peak is shifted to the phase $\sim -9.5^\circ$ from the meridional plane (M) which is at 0° .